

Topics : Work, Power and Energy, Center of Mass, Circular Motion

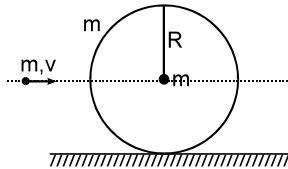
Type of Questions

Type of Questions	M.M., Min.
Single choice Objective ('-1' negative marking) Q.1 to Q.4	(3 marks, 3 min.) [12, 12]
Multiple choice objective ('-1' negative marking) Q.5 to Q.6	(4 marks, 4 min.) [8, 8]
Assertion and Reason (no negative marking) Q. 7	(3 marks, 3 min.) [3, 3]

1. A stone of mass M is tied at the end of a string, is moving in a circle of radius R , with a constant angular velocity ω . The total work done on the stone, in any half circle, is :

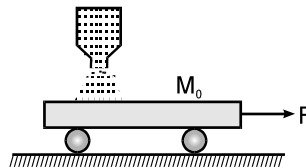
- (A) $\pi MR^2 \omega^2$ (B) $2 MR^2 \omega^2$ (C) $MR^2 \omega^2$ (D) 0

2. A hollow sphere of mass ' m ' and radius R rests on a smooth horizontal surface. A simple pendulum having string of length R and bob of mass m hangs from top most point of the sphere as shown. A bullet of mass ' m ' and velocity ' v ' partially penetrates the left side of the sphere and stick to it. The velocity of the sphere just after collision with bullet is.



- (A) $\frac{v}{2}$ (B) $\frac{v}{3}$ (C) $\frac{2v}{3}$ (D) $\frac{3v}{5}$

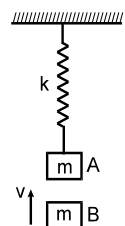
3. In the fig. shown a cart moves on a smooth horizontal surface due to an external constant force of magnitude F . The initial mass of the cart is M_0 and velocity is zero. Sand falls on to the cart with negligible velocity at constant rate μ kg/s and sticks to the cart. The velocity of the cart at time t is :



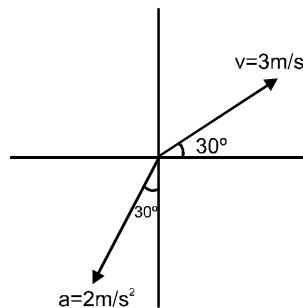
- (A) $\frac{F t}{M_0 + \mu t}$ (B) $\frac{F t}{M_0} e^{\mu t}$ (C) $\frac{F t}{M_0}$ (D) $\frac{F t}{M_0 + \mu t} e^{\mu t}$

4. Block 'A' is hanging from a vertical spring and is at rest. Block 'B' strikes the block 'A' with velocity ' v ' and sticks to it. Then the value of ' v ' for which the spring just attains natural length is:

- (A) $\sqrt{\frac{60 mg^2}{k}}$ (B) $\sqrt{\frac{6 mg^2}{k}}$
(C) $\sqrt{\frac{10 mg^2}{k}}$ (D) none of these



5. A strip of wood of mass M and length ℓ is placed on a smooth horizontal surface. An insect of mass m starts at one end of the strip and walks to the other end in time t moving with a constant speed.
- (A) the speed of insect as seen from the ground is $< \frac{\ell}{t}$
- (B) the speed of the strip as seen from the ground is $\frac{\ell}{t} \left(\frac{M}{M+m} \right)$
- (C) the speed of the insect as seen from the ground is $\frac{\ell}{t} \left(\frac{M}{M+m} \right)$
- (D) the total kinetic energy of the system is $\frac{1}{2} (m + M) \left(\frac{\ell}{t} \right)^2$.
6. Initial velocity and acceleration of a particle are as shown in the figure. Acceleration vector of particle remain constant. Then radius of curvature of path of particle :



- (A) is $9m$ initially
- (B) is $\frac{9}{\sqrt{3}} m$ initially
- (C) will have minimum value of $\frac{9}{8} m$
- (D) will have minimum value $\frac{3}{8} m$
7. **STATEMENT-1** : A sphere of mass m moving with speed u undergoes a perfectly elastic head on collision with another sphere of heavier mass M at rest ($M > m$), then direction of velocity of sphere of mass m is reversed due to collision [no external force acts on system of two spheres]
- STATEMENT-2** : During a collision of spheres of unequal masses, the heavier mass exerts more force on lighter mass in comparison to the force which lighter mass exerts on heavier mass.
- (A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
- (B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
- (C) Statement-1 is True, Statement-2 is False
- (D) Statement-1 is False, Statement-2 is True

Answers Key

DPP NO. - 56

1. (D) 2. (A) 3. (A) 4. (B)
 5. (A), (C) 6. (A), (C) 7. (C)

Hint & Solutions

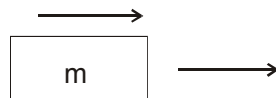
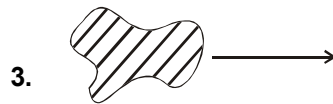
DPP NO. - 56

1. (Easy) Since there is no change in kinetic energy of stone, the total work done on stone in any duration is zero.
2. For the duration of collision the pendulum does not exert any force on the sphere in the horizontal direction.

Hence the horizontal momentum of bullet + sphere is conserved for the duration of collision. Let v' be the velocity of bullet and sphere just after the collision.

\therefore from conservation of momentum
 $(m + m) v' = mv$

or $v' = \frac{v}{2}$



Formula $F = m \frac{dv}{dt} + (V - u) \frac{dm}{dt}$

Here $u = \text{velocity of sand} = 0$
 $m = M_0 + \mu t = \text{mass at time } t$

and $\frac{dm}{dt} = \mu$

$\therefore F = (M_0 + \mu t) \frac{dv}{dt} + v \mu$

$(F - \mu v) dt = (M_0 + \mu t) dv$

$$\int_0^t \frac{dt}{M_0 + \mu t} = \int_0^v \frac{dv}{F - \mu v}$$

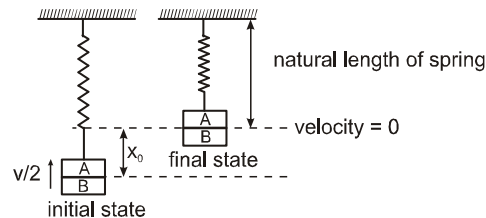
$$\frac{1}{\mu} \log(M_0 + \mu t) \Big|_0^t = \frac{1}{\mu} [\log(F - \mu v)] \Big|_0^v$$

$$\log \frac{M_0 + \mu t}{M_0} = \log \left(\frac{F}{F - \mu v} \right)$$

$$F - \mu v = \frac{M_0 F}{M_0 + \mu t} \Rightarrow v = \frac{Ft}{M_0 + \mu t}$$

4. (B) The initial extension in spring is $x_0 = \frac{mg}{k}$
 Just after collision of B with A the speed of combined mass is $\frac{v}{2}$.

For the spring to just attain natural length the combined mass must rise up by $x_0 = \frac{mg}{k}$ (see fig.) and comes to rest.



Applying conservation of energy between initial and final states

$$\frac{1}{2} 2m \left(\frac{v}{2}\right)^2 + \frac{1}{2} k \left(\frac{mg}{k}\right)^2 = 2mg \left(\frac{mg}{k}\right)$$

Solving we get $v = \sqrt{\frac{6mg^2}{k}}$

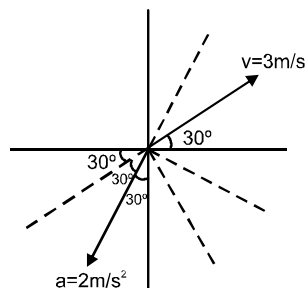
5. From linear momentum conservation
 v & v' are speed of strip and insect w.r.t. ground
 $Mv = mv'$

$$v + v' = \frac{\ell}{t} \Rightarrow v' \left(1 + \frac{m}{M}\right) = \frac{\ell}{t}$$



$$v' = \left(\frac{M}{m+M}\right) \frac{\ell}{t}$$

6. Initially $ROC = \frac{v^2}{a \sin 30^\circ} = \frac{9}{1} \text{ m}$



For minimum $ROC = \frac{(v \sin 30^\circ)^2}{a} = \frac{9}{8} \text{ m}$.

7. Statement-2 contradicts Newton's third law and hence is false.

